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Smoothing Top Height Estimates from Two Lodgepole Pine Height Models

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Abstract

The two models for estimating the top height of lodgepole pine (*Pinus contorta* var. *latifolia* Dougl.) are designed to function over different age ranges: a juvenile height model for total ages below 15 and a height-age model above breast height age ½. Four options are available to splice these models together to give a smooth height trajectory near the splice point. The option resulting in the smoothest trajectory uses the juvenile height model below breast height age 0 and the height-age model above breast height age 2, and linearly interpolates heights between breast height age 0 and 2.

Introduction

Two models are available for estimating the top height of lodgepole pine (*Pinus contorta* var. *latifolia* Dougl.) in the interior of British Columbia: a juvenile height model (Nigh and Love 1999) and a height-age model (J.S. Thrower and Associates Ltd. 1994). The juvenile height model estimates top height from germination up to total age 15. The height-age model estimates top height for trees older than breast height age ½. Both models can be used together to get height estimates from germination up to almost any age. However, the height estimates should

be smoothed to give a consistent height trajectory.

Background

The juvenile height model (equation 1) uses site index to estimate top height from germination (total age 0) to total age 15. The functional form of the model is a Type I combined exponential and power function (Sit and Poulin-Costello 1994), which is constrained to give a height estimate of 0 m at age 0; that is:

$$H = (-0.03993 + 0.004828 \times SI) \times A_t^{1.902} \times 0.9645^{A_t} \quad [1]$$

where:

H = top height (m),
 SI = site index (m at breast height age 50), and
 A_t = total age (years).

For trees older than breast height age ½, the height-age model (equation 2) estimates top height from the site index. The functional form of the model is a constrained log-logistic function that goes through breast height (1.3 m) at breast height age ½ and through the site index at breast height age 50; that is:

$$H = 1.3 + (SI - 1.3) \times \frac{1 + e^{7.6298 - 1.3563 \times \ln(49.5) - 0.8949 \times \ln(SI - 1.3)}}{1 + e^{7.6298 - 1.3563 \times \ln(A_b - 0.5) - 0.8949 \times \ln(SI - 1.3)}} \quad [2]$$

where:

- H = top height (m),
 SI = site index (m at breast height age 50), and
 A_b = breast height age (years).

Although the juvenile height model is based on total age, it can be converted to breast height age. This is done by subtracting the number of years it takes the trees to reach breast height age 0 from the total age. However, this does not constrain the model to go through 1.3 m at breast height age $\frac{1}{2}$, as is the case for the height-age model. Therefore, the juvenile height model (after it is converted to breast height age) and the height-age model will not necessarily give the same height estimates at breast height age $\frac{1}{2}$, or at any other age. This creates irregularities in the height trajectories when the shift is made from model (1) to model (2). Consequently, the height estimates from these models should be smoothed to reduce or remove the irregularity.

If models (1) and (2) estimate the same attribute (i.e., height) over different age ranges, why do they need to be spliced together and why do they not give the same height estimates at breast height ages 1 and older? There are several reasons for this discrepancy. For instance, because the models were developed from different, independent data sets, the height trajectories of the plots in the two data sets will be different, and therefore the models developed from these data will have different height trajectories. If both models were developed from the same data, it would still be unlikely that they would join smoothly because of different model functions.

Smoothing Procedure

Four options were considered to smooth the height estimates:

1. Estimate heights below breast height age $\frac{1}{2}$ with the juvenile height model and estimate heights

above breast height age $\frac{1}{2}$ with the height-age model.

2. Estimate heights up to total age 15 with the juvenile height model and estimate heights above total age 15 with the height-age model.
3. Estimate heights for trees up to breast height age 0 with the juvenile height model and estimate heights for trees older than breast height age 2 with the height-age model. Between breast height age 0 and 2, the heights from the two models are averaged.
4. Estimate heights for trees up to breast height age 0 with the juvenile height model and estimate heights for trees older than breast height age 2 with the height-age model. Heights for breast height ages between 0 and 2 are linearly interpolated from the breast height age 0 and breast height age 2 height estimates.

These four methods of smoothing height estimates were compared graphically by plotting the top height trajectories for different site index levels (12, 15, 18, 21, and 24 m). The option that gave the smoothest looking trajectories was chosen.

Results

Figures 1, 2, 3, and 4 show the height trajectories that correspond to smoothing options 1, 2, 3, and 4, respectively.

Discussion

Option 4 is recommended because it resulted in the smoothest height trajectories. This option essentially generates three models: the juvenile height model up to breast height age 0; a linear interpolation model between breast height ages 0 and 2; and the height-age model above breast height age 2.

There are legitimate reasons why the juvenile height model (1) and the height-age model (2) give different height estimates for a given age and site index. However, one model cannot be deemed correct and the other invalid. In fact, except for the occasional site, both models are inexact and will probably be inexact even when used to estimate the average height trajectory for a given level of site index.

If both models were developed at the same time, an attempt to smooth

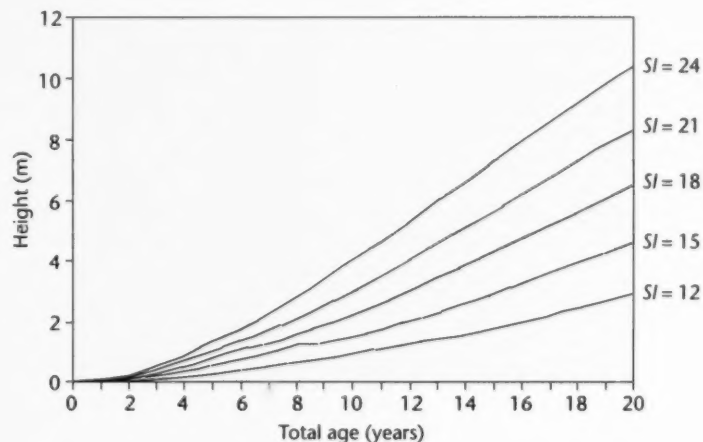


FIGURE 1 Option 1: Smoothing by joining height estimates at breast height for site index (SI) 12, 15, 18, 21, and 24 m.

FIGURE 2 Option 2: Smoothing by joining height estimates at total age 15 for site index (SI) 12, 15, 18, 21, and 24 m.

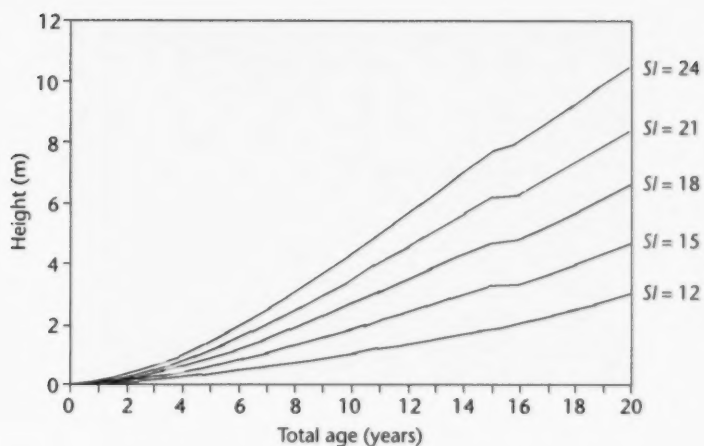


FIGURE 3 Option 3: Smoothing by averaging height estimates between breast height ages 0 and 2 for site index (SI) 12, 15, 18, 21, and 24 m.

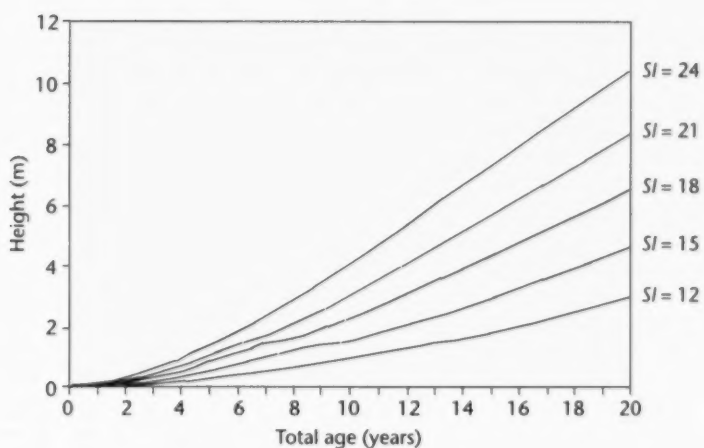
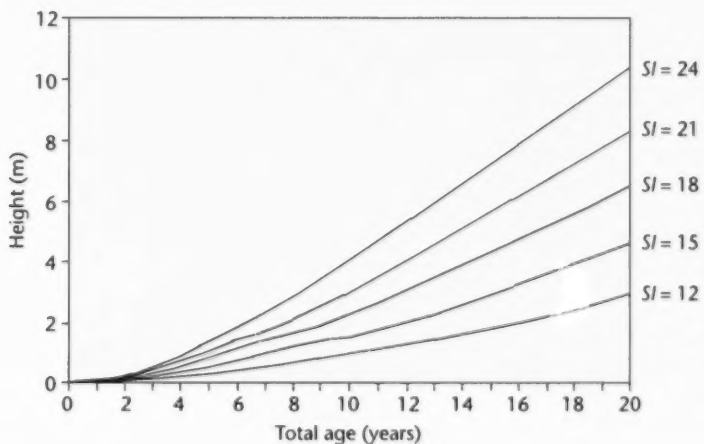


FIGURE 4 Option 4: Smoothing by linearly interpolating height estimates between breast height ages 0 and 2 for site index (SI) 12, 15, 18, 21, and 24 m.



the height trajectories could be made by splining the models; that is, by forcing them to join at a pre-specified point and equating their first derivatives at the join point to smooth the function (Burden et al. 1978). However, putting these constraints on the models may result in a poor fit to the data, and may require the use of different, and perhaps poorer-fitting, functions.

The juvenile height model and the height-age model can be used independently for some applications. For example, the juvenile height model may be used alone when:

1. height is being estimated only up to total age 15 or less;
 2. years to breast height are being determined; or
 3. green-up age is being found.
- The height-age model can be used alone when height estimates only above breast height are required. The spliced version of the models should be used in situations where height trajectories that cross the age ranges of the two models are required and smooth trajectories are favoured.

Conclusion

Several options are available to smooth height estimates from a

juvenile height model and a height-age model. The best option, based on a visual assessment, is to estimate heights below breast height age 0 with the juvenile height model, estimate heights above breast height age 2 with the height-age model, and linearly interpolate heights between breast height ages 0 and 2. Both models can be used without smoothing for some applications.

References

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